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Analysis of Whispering Gallery Modes of an Optical Pillbox Resonator using Finite Element BPM (FE-BPM)

Shirin Akter[1], Nahid Akter[2] and Anis Ahmed[3]

[1]Atomic Energy Centre, Dhaka-1000, Bangladesh.
[2]Dept. of Computer Science & Telecommunication Engineering, Noakhali Science & Technology University, Noakhali, Bangladesh.
[3]Dept. of Applied Physics, Electronics and Communication Engineering, University of Dhaka, Bangladesh.

Abstract

To deal with the radiation from an axially symmetric graded index optical pillbox resonator embeded in $LiNbO_3$ substrate formed by proton exchanged method, an eigenvalue equation is derived using finite-element beam propagation method (FE-BPM) in a cylindrical coordinate system. By solving the eigenvalue equation, the resonant whispering gallery modes and the angular phase constant of resonator are evaluated. In order to fine an optimized characteristics of the resonator, the core radius is taken a=200 μm and a resonant wavelength λ_0= 0.900115 μm. At this wavelength n_{co} and n_{cl} of the proton exchange waveguide in $LiNbO_3$ are found to be 2.293 and 2.165, respectively. Using these values a step-index resonator is analyzed and the obtained results are found in good agreement with the analytical ones. The analysis is carried out for uniform sampling space with sampling point M = 551. The resonant characteristics of an optical resonator at wavelength 1.55 μm is also calculated. With increase of wavelength it is found that the number of resonant whispering gallery modes decreases. In order to get a mono-mode resonator, the radius of the resonator should be tailored and take a tradeoff between the size and the desired mode number.

1. Introduction

In optical Integrated Circuits (OIC), the modulated light is guided by thin-film optical waveguides. A hybrid OIC is made of three different materials such as compound semiconductor (e.g. GaAlAs), dielectric (e.g. Glass or $LiNbO_3$) and semiconductor (e.g. Si) to be used as source, waveguide and detector, respectively. The sources and detectors are usually called active devices while the waveguide is called passive device. In a Whispering Gallery mode resonator the materials can be used to make a passive resonator such as wavelength filter or an active resonator such as laser. A schematic representation of such a pillbox resonator and its coordinate axis are shown in Figure. 1.

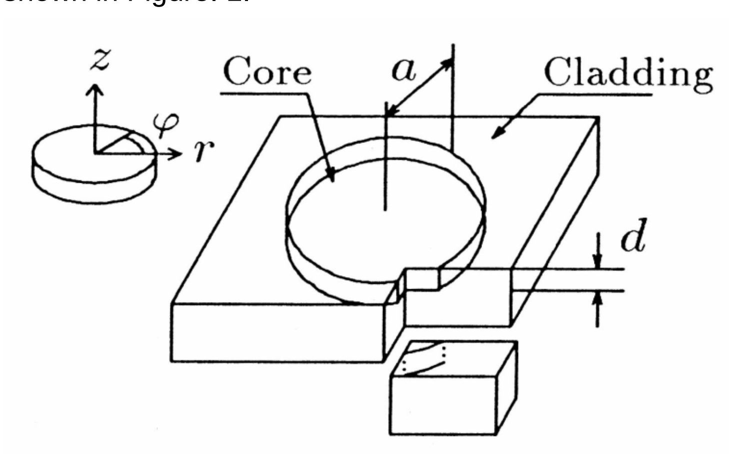


Fig. 1. Configuration of pillbox resonator

For low power semiconductor laser Fabry-Perot resonator arrangement is suitable. But for a high power semiconductor laser, Fabry-Perot resonator does not last long. As the light impinges on the mirror perpendicularly, the surface of the mirror will become rough with time of use and finally the resonator looses its power. On the other hand, if we can generate a light wave inside the circular pillbox core, it will remain confined there by multiple total internal reflections on the circular boundary as shown in Figure.2. For a high power laser such resonator-structure is assumed to be good, because this time light impinges on the circular-core boundary obliquely which reduces the risk of surface damage. Therefore, one of our research goal is to analyze a pillbox resonator to determine its resonant wavelengths and resonant modes in order to use it in optical integrated circuits (OICs).

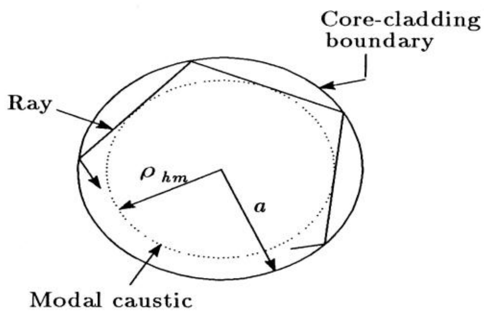


Fig. 2. Top view of the pillbox resonator with whispering gallery Modes

2. Eigenmode analysis of lossless pillbox using FE-BPM formulation

It was Mahapatra et al. who made race-trace resonator in $LiNbO_3$ by the proton exchange method [1]. In order to use large electrooptic coefficient of $LiNbO_3$, we assume a proton exchanged pillbox resonator in $LiNbO_3$.For the analysis of step index pillbox resonator, many approximate methods have been reported [2]. But these methods are not applicable if the index distribution of pillbox resonator is arbitrary. M. Matsuhara used the Galerkin method to analyze the propagation characteristics of lightwave of dielectric waveguides. This method is called the finite element beam propagation method (FE-BPM) [3]. To analyze an axially symmetric pillbox resonator with arbitrary index profile, a numerical method based on the FE-BPM in cylindrical coordinate system is developed. In our numerical method, an eigenvalue equation is derived from the expression of the FE-BPM [4]. In our analysis, we assume an axially symmetric pillbox resonator. Since the WGMs of the pillbox resonator cling to the edge of the pillbox resonator, the calculated region is taken from $r = r_s$ to $r = r_e$. These two boundaries are named numerical boundaries as shown in Fig 3.

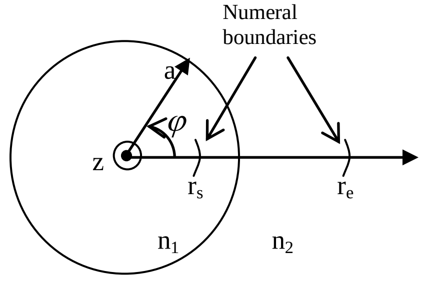


Fig. 3. Pillbox resonator showing the calculated region bounded by the numerical boundaries

The fields of the 2- D pillbox resonator are thought to be uniform in the z-direction. The field components E_z and H_z for TE and TM modes, respectively, are parallel to the core-cladding boundary along the circumference of the pillbox [5]. The other nonzero field components of the TE modes are H_r and H_φ while that of the TM modes are E_r and E_φ. The WGH modes will correspond to the TE modes of 2-D pillbox if the field components E_r and E_φ of WGH modes, which are relatively weak, are neglected. Here the analysis of the TE modes is described while the analysis of TM modes will be similar.Let us suppose that electric field component $E_z(r,\varphi)$ propagates inside a lossless pillbox resonator in the $+\varphi$ direction as

$$E_Z(r,\varphi) = t_m(r)\exp(-jh\varphi) \qquad (1)$$

where $t_m(r)$ is the radial eigenmode field distribution and h is the angular phase constant. Note that h is now called differently . When a propagating lightwave completes one round trip inside pillbox, resonance will occur if the total phase shift becomes an integer multiple of 2π , i.e,

Total phase shift in one round trip= 2π p...(2)

where p is the azimuthal mode number, $p = 0,\pm1,\pm2,..$. So the real part of $\tilde{h}$ in Eq. (1) will be equal to p which is an integer. Note that h will be an integer at resonance.

3. Propagation mode of pillbox resonator

If a guided mode propagates along the circumference of the pillbox resonator, h is given by $h = k_0 a_{neff}$ where $k_0 = 2\pi/\lambda_0$,is the free space wavenumber, a is the radius of the pillbox and n_{eff} is the effective index of the guided mode. The effective index n_{eff} has values between n_2 and n_1 $(n_2 < n_1)$. If we write $N_m = n_2 + n_\delta$, then angular phase constant h can be expressed as

$$h \equiv h_0 + h_\delta; \quad h_0 = k_0 a n_2 \quad h_\delta = k_0 a n_\delta \qquad (3)$$

In the above equation h_0 is a given constant for some reference value of the angular phase constant. We take h equal to angular phase constant of the cladding region. Using Eq.(3) we can write Eq. (1) as

$$E_Z(r,\varphi) = \phi(r,\varphi)\exp(-jh_0\varphi) \qquad (4)$$

where $\phi(r,\varphi) = t_m(r)(\exp(-jh_\delta\varphi)$ ....(5)

Substituting Eq. (4) into scalar Helmholtz equation $\nabla^2\phi + k_0{}^2 n^2 \phi = 0$ in cylindrical coordinate system, we get $(\partial/\partial z = 0)$

$$\frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) - 2j\frac{h_0}{r^2}\frac{\partial\phi}{\partial\varphi} + \left(k_0{}^2 n^2(r) - \frac{h_0{}^2}{r^2}\right)\phi = 0 \qquad (6)$$

where n(r) is axially symmetric index distribution of the pillbox. Assuming that the index change over the distance of one wavelength is very slow, Eq. (6) can be modified to the scalar fresnel equation as [4]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi'}{\partial r}\right) - 2j\frac{h_0}{r^2}\frac{\partial\phi'}{\partial\varphi} + \left(k_o{}^2 n^2(r) - \frac{h_0{}^2}{r^2}\right)\phi' = 0 \qquad (7)$$

In the above equation a superscript prime is used to discriminate the parameters of Fresnel equation from those of Helmholtz equation. Let us consider that the eigensolution of Eq. (7) is

$$\phi'(r,\varphi) = t'_m \exp(-jh'_\delta\varphi); \quad h'_\delta = k_0 a n'_\delta \qquad (8)$$

where n'_δ and h'_δ have values close to n_δ and h_δ , respectively. For the analysis of optical fibers, Feit et. al. derived two expressions relating the parameters of Fresnel equation to those of

Helmholtz equation [6]. In our case, the following two relations are found by substituting Eq.(5) into Eq.(6) and Eq. (8) into Eq. (7) $t'_m(r) = t_m(r)$ ....(9)

$$h'_\delta = \frac{{h_\delta}^2 + 2h_0 h_\delta}{2h_0}$$ ....(10)

From Eqs.(9) and (10) we see that it is enough to solve Eq.(7) instead of Eq. (6), because the eigenmode field distribution obtained from Eq. (6) is the same as that obtained from Eq. (7) and the eigenvalues obtained from Eqs. (7) and (6) are related by Eq. (10).

Now from Eq. (8) we get

$$\frac{\partial \phi'}{\partial \varphi} = -jh'_\delta \phi'$$ ....(11)

Substituting Eqs. (8) and (11) into Eq.(7), we obtain

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dt'_m}{dr}\right) - \frac{2h_0 h'_\delta t'_m}{r^2} + \left({k_0}^2 n^2(r) - \frac{{h_0}^2}{r^2}\right)t'_m = 0$$ ....(12)

In order to solve Eq. (12) by Galerkin method, the radial field distribution $t'_m(r)$ is expanded as

$$t'_m(r) = \sum_{i=1}^{M} f_i u_i(r)$$ ……………..(13)

where $u_i(r)$ is the known basis function, f_i is unknown expansion coefficient and M the number of sample points. By applying the Galerkin method along with the Dirichlet boundary condition, Eq. (12) can be written as

$$\int_{rs}^{re}\left[\left(r{k_0}^2 n^2(r) - \frac{{h_0}^2}{r}\right) - \frac{2h_0 h'_\delta}{r}\right]u_i t'_m dr - \int_{rs}^{re}\left[r\frac{dt'_m}{dr}\frac{du_i}{dr}\right]dr = 0$$ ....(14)

where the numerical integration is performed in the radial direction from r_s to r_e .

The index distribution n(r) of the pillbox resonator is now expressed

$$n(r) = n_2 + \Delta g(r)$$ ....(15)

where Δ is the maximum index difference between the core and the cladding, and g(r) is the index variation function which is arbitrary.

For the propagation modes of the pillbox, we assume that the wavelength λ_0 of the lightwave is known, but the angular phase constant h and the radial eigenmode field distribution $t_m(r)$ of each mode are to be calculated. Equation (14), in combination with Eq. (13), is arranged so that h'_δ appears as the eigenvalue of the eigenvalue equation

$$h'_\delta \sum_{j=1}^{M} a_{ij} f_i = \sum_{j=1}^{M} b_{ij} f_i,$$ (i =1, 2,.....M)..(16)

Where

$$a_{ij} = 2h_0 \int_{rs}^{re} \frac{u_i(r) u_j(r)}{r} dr$$ …………...……………….......(17)

$$b_{ij} = \int_{rs}^{re}\left[r{k_0}^2 n^2(r) u_i(r) u_j(r) - {h_0}^2 \frac{u_i(r) u_j(r)}{r} - r\frac{du_i(r)}{dr}\frac{du_j(r)}{dr}\right]dr$$ …...…(18)

Dividing both side of equation (5.3-14) by $k_0 a\Delta$, we get the following eigenvalue equation in matrix form

$$VF = QF \quad \text{..........} (19)$$

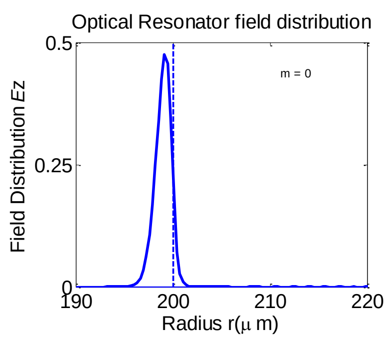
where Q and V is given by $\quad Q = \frac{1}{k_0 a \Delta} A^{-1} B$ and $V = \frac{h'_\delta}{k_0 a \Delta}$ ....(20)

The elements of matrices A and B are given in Eqs. (17) and (18), respectively. The numerical calculation of A and B are given in section 5.4. In Eq. (19), F is the eigenvector and V corresponds to the eigenvalue. By using the relation of h'_δ from Eq. (8), it is possible to show that the eigenvalue V may vary from 0 to 1 for the guided modes.

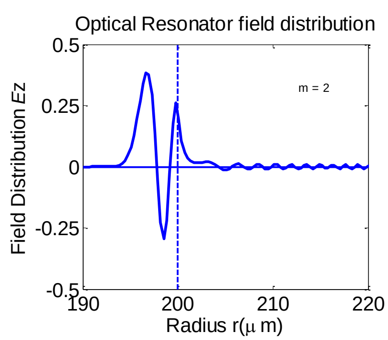
Equation (19) is thus the required equation to calculate $t'_m(r)$ and V for the propagation modes of the pillbox resonator. Substituting the value of V into Eq. (20), we get h'_δ.The obtained field distribuation $t'_m(r)$ and the coorresponding phase constant h'_δ of the fresnel equation are then converted into those of Helmholtz by using Eqs. (9) and (10). The angular phase constant h is then found from Eq. (3).

4. Results and discussion

Let us first consider the case of the fundamental radial mode (m=0). Since we are interested in the wavelength near 0.90 μm, λ_0 is taken 0.900115 μm , which is the resonant wavelength of the radial mode m=0. The angular phase constant h'_δ is calculated by solving Eq. (19). Equation (10) is used to convert h'_δ of Fresnel equation into h_δ of Helmholtz equation.Similarly λ_0 is selected at the resonant wavelengths of other radial modes (m=1, 2,...) and the corresponding h'_δ is calculated. In table 1, the numerical values of h (as real variable) are shown. In calculating the angular phase constant h'_δ , the radial field distribution $t'_m(r)$ of the pillbox is also found. For a pillbox resonator with radius a =200 μm, five radial modes are found. In Eq. (9) it is shown that the eigenmode field distribution $t'_m(r)$ of the Fresnel equation is the same as $t_m(r)$ of Helmholtz equation.



(a)



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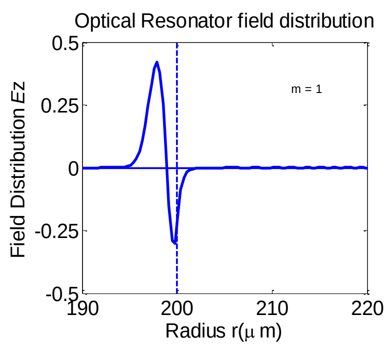
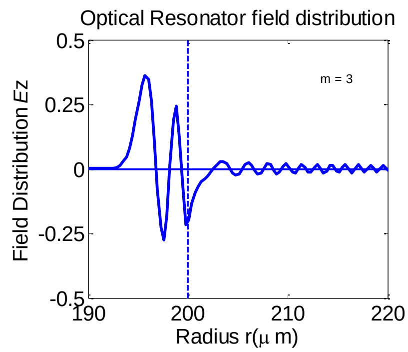
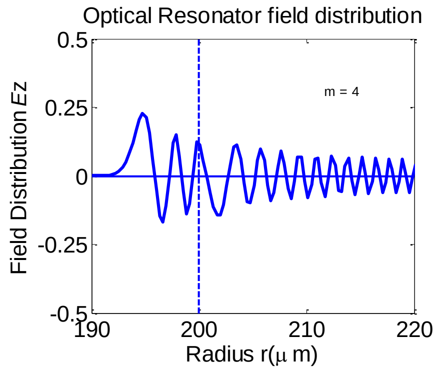


(c)



(d)



(e)

Fig. 4: Eigenmode field distributions for (a)m=0; (b) m=1; (c) m=2; (d) m=3; (e) m=4.

From Figure 4 (a) and (b) we see that most of the power is confined in the core region and there is no radiation outside the core region. In Fig 4(c) and (d) we see that there is slight radiation in the cladding region. The existence of fixed numerical boundaries on both side of the calculated region is also responsible for the radiation of the field distributions. These virtual boundaries cause the radial fields to be reflected back. Since the radiated fields are more for higher order radial modes, the radiation of the field distributions is found large for m= 4 which is shown in Fig. 4 (e).

Table 1: Numerically calculated angular phase constant (h) of different modes for different light wavelength (λ_0).

λ_0= 0.899940		λ_0= 0.900047		λ_0 =.900115		λ_0 =1.55	
m	h	m	h	m	h	m	h
0	2942.8	0	2942.5	0	2942.3	0	1713.6
1	2965.0	1	2964.6	1	2964.4	1	1731.1
2	2982.9	2	2982.6	2	2982.4	2	-
3	2998.5	3	2998.2	3	2997.9	3	-
4	3011.7	4	3011.5	4	3011.3	4	-
5	3013.1	5	3012.9	5	3012.7	5	-

From Table 1 we see that for increasing the value of angular phase constant h the light wavelength is decreases. We know that $h = \frac{2\pi a n_{eff}}{\lambda_0}$. So our results satisfy this condition.

Conclusion

For the analysis of the light wave propagation through circular optical resonator by the BPM, one must know the initial field distribution of the pillbox resonator. But for the analysis by FE-BPM, no initial field distribution is necessary. Thus our numerical method is a very useful tool for the analysis of an axially symmetric pillbox resonator with arbitrary index profile.

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