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Estimation of Timing Offset Error in OFDM System and Selecion of Modulation Technique for Best Performance

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Abstract

This paper proposes an idea of developing a model of estimating timing offset error of OFDM (Orthogonal Frequency Division Multiplexing) system without the use of additional pilots relying on inherent characteristics of the OFDM signal. An analytical expression has been derived and formulated to analyze the effect of bit error rate (BER) due to timing offset introduced by the transmission channel. This will help in determining the exact length of cyclic prefix to be added to each OFDM symbol to avoid misinterpretation by the receiver. The performances have also been evaluated under coded (convolution) and uncoded systems. The introduction of channel coding decreases this basic impairment of OFDM systems significantly. Simulated results show that the symbol error rate (SER) linearly depends on timing offset. Finally, using Matlab simulation a comparison among the three modulation techniques is done and found that BPSK gives the best bit error rate performance at the present of timing offset error. It is expected that further works on the proposed estimated method will lead to a standard model so that at the receiving end the effect of timing offset can be eliminated totally.

Keywords: OFDM; IFFT; FFT; AWGN; Timing Offset; BER; SER; Convolution coding

1. INTRODUCTION

The need for OFDM (Orthogonal Frequency Division Multiplexing) system came from the idea of efficient use of spectrum as well as bandwidth where the data transmission becomes four times faster than the present one. OFDM supports the technologies like DAB (Digital Audio Broadcasting) or DVB (Digital Video broadcasting) [1]. It is a special case of multicarrier transmission, where a single data stream is transmitted over a number of lower rate subcarriers. All the subcarriers within the OFDM signal are time and frequency synchronized to each other, allowing the interference between subcarriers to be carefully controlled. The OFDM systems are sensitive to symbol timing error, frequency offset and timing offset. The demodulation of a signal with an offset in the arrival timing period will cause higher Bit Error Rate (BER) and degrade the performance of a symbol synchronizer [2]. The timing offset error occurs due to the delay in the reception of the information causing some interference of information for overlapping of consecutive OFDM symbols. This paper particularly focuses on estimation of timing offset present in the symbol so that the receiver can accurately interpret the information that is sent by the transmitter. The main work of the paper involves the maturity of a standard equation to estimate timing offset.

2. OFDM System Description

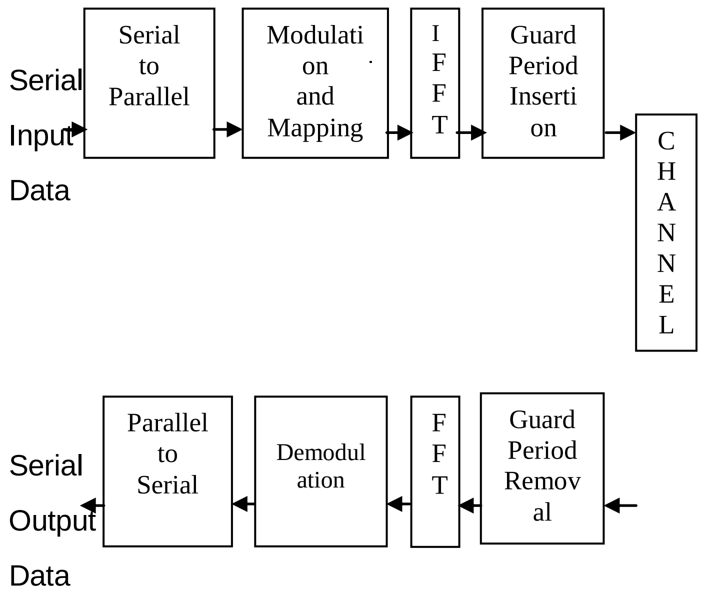

Figure 1. Block diagram of OFDM system model

Fig. 1 shows basic steps of an OFDM system. At first the serial input data are converted into parallel data by the Serial to Parallel converter. Then by modulation and mapping these parallel bits are assigned on to a desired number of orthogonal subcarriers. In the IFFT stage, all subcarriers with data bits are then converted to frequency domain and construct an OFDM symbol then a guard period is inserted into the symbol. Then the transmitter send this OFDM symbol stream to the receiver and the reverse process occurs in the receiving end. In OFDM, the incoming data stream is grouped into blocks of N data symbols by the stage serial to parallel and modulation and mapping. These groups are called OFDM symbols and are represented by a vector-

$$X_m = [X_{0,m}, X_{1,m}, X_{2,m}, \ldots\ldots X_{n-1,m}]^T$$

Next an IDFT operation is performed on each data symbol blocks and a cyclic prefix of length N_{cp} is added. The resulting complex baseband discrete time signal of the mth OFDM symbol is-

$$s_m(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{k,m} \cdot e^{j2\pi k(n-N)/N_{cp}}$$

where, $k \in [0, N + N_{cp} - 1]$ and denotes the timing index [3]. Therefore, the complete time signal $S(k)$ is given by-

$$S(k) = \sum_{m=0}^{\infty} S_m(k - m(N + N_{cp}))$$

In general the received signal is the sum of a linear convolution with the discrete channel impulse response $h(\lambda)$ and the complex additive white Gaussian noise (AWGN) $n(k)$. Therefore, the received signal $R(k)$ is-

$$R(k) = \sum_{\lambda=0}^{N_{cp}-1} h(\lambda) \cdot S(k-\lambda) + n(k)$$

Now, for simplicity in the calculation assuming that the transmitted signal $S(k)$ is only affected by AWGN and the channel is modeled as a delay in the impulse response $\delta(k-\theta)$, where θ is the integer-valued unknown arrival time offset of a symbol. Thus, the received signal becomes-

$$R(k) = S(k) * \delta(k-\theta) + n(k)$$

$$= S(k-\theta) + n(k)$$

3. BASICS Of OFDM TIMING OFFSET

The delay in the reception of the OFDM symbol due to the transmission channel is called timing offset of OFDM system. If the timing offset is smaller than the length of the guard interval i.e. cyclic prefix, then the orthogonality of the carriers is not hampered and no overlapping of symbol occurs. Even though the cyclic prefixes are added to each symbol for preventing disruption of information, OFDM receiver misinterprets the sent information for overlapping of consecutive symbols when timing offset is longer than cyclic prefix. Each offset, positive or negative causes the decrease of the relevant signal due to the diminished number of real data in the window being processed [4].

4. ANALYTICAL EXPRESSION OF TIMING OFFSET

Using subcarriers OFDM system divides frequency selective fading channel info into multiple sub channel each of which exhibits flat fading. Again, the power spectrum of additive white Gaussian noise is also flat in nature. Therefore it is assumed that the received OFDM symbol follows complex Gaussian distribution. Now, consider that in an OFDM system, $S(k)$ is the transmitted OFDM symbol, $R(k)$ and $R'(k)$ are the received OFDM symbol with and without timing offset. According to (1), the received signals are-

$$R'(k) = S(k) * \delta(k-\theta) + n(k)$$

$$= S(k-\theta) + n(k)$$

and,

$$R(k) = S(k) * \delta(k) + n(k)$$

$$= S(k) + n(k)$$

Now, the probability density function (pdf) of the received OFDM symbols are-

$$f(R'(k)) = \frac{1}{\sqrt{2\pi}\sigma_{s1}} e - \frac{|R'(k)|^2}{2\,\sigma_{s1}^2}$$

and,

$$f(R(k)) = \frac{1}{\sqrt{2\pi}\sigma_{s_2}} e - \frac{|R(k)|^2}{2\,\sigma_{s2}^2}$$

here, $f(\cdot)$ denotes probability density function of a Gaussian distribution [8], and σ_{s1} and σ_{s_2} are the variances of the received OFDM symbol with and without timing offset.

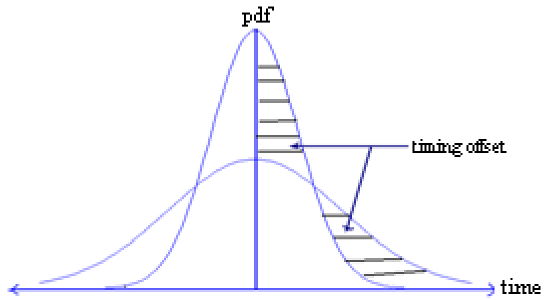

Figure 2. Probability density function of two received OFDM symbol (with and without timing offset)

Therefore, the probability of OFDM symbol error,

$$P_e = \int_0^{\infty} [f(R'(k)) - f(R(k))]dk$$

$$= \frac{1}{\sqrt{2\pi}\sigma_{s1}} \int_0^{\infty} e - \frac{|R'(k)|^2}{2\sigma_{s1}^2} dk - \frac{1}{\sqrt{2\pi}\sigma_{s2}} \int_0^{\infty} e - \frac{|R(k)|^2}{2\sigma_{s2}^2} dk$$

$$= \frac{1}{\sqrt{2\pi}\sigma_{s1}} \int_0^{\infty} e - \frac{|S(k-\theta)|^2}{2\sigma_{s1}^2} dk - \frac{1}{\sqrt{2\pi}\sigma_{s2}} \int_0^{\infty} e - \frac{|S(k)|^2}{2\sigma_{s2}^2} dk$$

here, $n(k)$ is neglected for simplicity in calculation. Now, take

$$I_1 = \frac{1}{\sqrt{2\pi}\sigma_{s1}} \int_0^{\infty} e - \frac{|S(k-\theta)|^2}{2\sigma_{s1}^2} dk$$

and,

$$I_2 = \frac{1}{\sqrt{2\pi}\sigma_{s2}} \int_0^{\infty} e - \frac{|S(k)|^2}{2\sigma_{s2}^2} dk$$

hence, $P_e = I_1 - I_2$ (2)

Let, $k - \theta = p$, so, $dk = dp$. Therefore, if $k = 0$ then $p = -\theta$ and, if $k = \infty$, then $p = \infty$. Now,

$$I_1 = \frac{1}{\sqrt{2\pi}\sigma_{s1}} \int_{-\theta}^{\infty} e - \frac{|S(p)|^2}{2\sigma_{s1}^2} dp$$

$$I_1 = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma_{s1}} \int_{-\theta}^{\infty} e - \frac{|S(p)|^2}{2\sigma_{s1}^2} dp$$

$$= Q(-\frac{\theta}{\sigma_{s1}})$$ [6];

here, $Q(\cdot)$ denotes Q-function where $Q(x)$ is defined as the probability that a standard normal random variable (zero mean, unit variance) exceeds x. Similarly, it can be got that, $I_2 = Q(0)$, therefore, according to (2),

$$P_e = Q(-\frac{\theta}{\sigma_{s1}}) - Q(0)$$

$$= [1 - Q(\frac{\theta}{\sigma_{s1}}) - Q(0)]$$

$$= [\frac{1}{2} - Q(\frac{\theta}{\sigma_{s1}})] \text{; since } Q(0) = \frac{1}{2}$$

$$= [\frac{1}{2} - \frac{1}{2} erfc(\frac{\theta}{\sigma_{s1} \times \sqrt{2}})]$$

Hence, the probability of OFDM symbol error is-

$$P_e = \frac{1}{2}[1 - erfc(\frac{\theta}{\sigma_{s1} \times \sqrt{2}})]$$

5. MODULATION TECHNIQES

Digital modulation techniques use a finite number of distinct symbols to represent digital data. In this paper Phase-shift keying (PSK), a digital modulation technique is studied and three popular types of PSK- Binary phase-shift keying (BPSK), Quadrature phase-shift keying (QPSK), and 8-PSK are analyzed and compared in OFDM environment through Matlab simulation. PSK uses phases to represent binary digits and each phase encodes an equal number of bits. Each pattern of these bits forms the symbol that in particular phase. The demodulator determines the phase of the received signal and maps it back to the symbol it represents. In Binary Phase Shift Keying changing the bit changes the sign of the transmitted signal and shifts the phase by π. Quadrature Phase Shift Keying (QPSK) is a form of Phase Shift Keying in which two bits are modulated at once, selecting one of four possible carrier phase shifts (0, Π/2, Π, and 3Π/2). QPSK perform by changing the phase of the In-phase (I) carrier from 0° to 180° and the Quadrature-phase (Q) carrier between 90° and 270°. This is used to indicate the four states of a 2-bit binary code. Each state of these carriers is referred to as a Symbol. 8-PSK is a form of phase modulation which is accomplished by the use of a discrete number of states. 8PSK refers to PSK with 8 sates. [7]

6. RESULTS AND DISCUSSIONS

Simulations have been conducted for an OFDM system with subcarriers=200, IFFT bin length = 1024, number of carriers = 200, bits per symbol = 2, symbol per carrier = 50. Here, BPSK is used as modulation scheme. The carriers are placed in such a way that the carriers and conjugate carriers form a mirror image of each other with guard period in between. The simulation is done according to how a real OFDM system works.

Fig. 3 shows the relationship between Symbol Error Rate (SER) and the SNR or E_b/N_o based on the derived analytical equation. The Symbol Error Rate increases with the decrease in the SNR (E_b/N_o). Symbol Error Rate is comparatively less for coded system as channel coding reduces the amount of noise addition in the channel.

Fig. 4 illustrates that E_b/N_o decreases with the increase in BER and the BER is higher when timing offset is present and increases with higher timing offset. Therefore, when there is more timing offset then it produces higher BER and thus results in lower Signal to Noise Ratio i.e. more noise.

The curve in the figure Bit Error Rate vs. E_b/N_o for a channel with convolution coding (fig. 5) reveals the same relationship as it is seen in the previous case and the introduction of convolution coding degraded the range of BER.

It is seen from the figure Symbol Error Rate (SER) vs. Timing Offset (fig. 6) that the SER is increasing with timing offset and the use of convolutional coding (channel coding) results lower in SER. Therefore, use of better channel coding such as Turbo coding will give better performance in degrading SER.

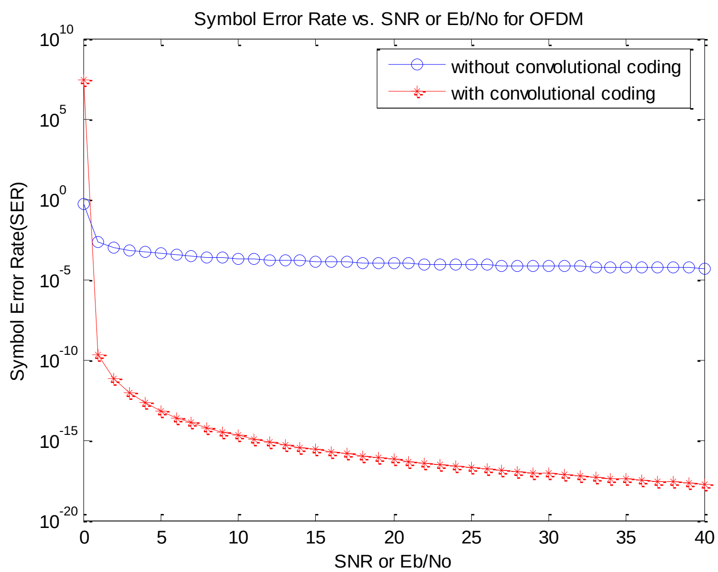

Figure 3. Symbol Error Rate vs SNR or E_b/N_o for OFDM

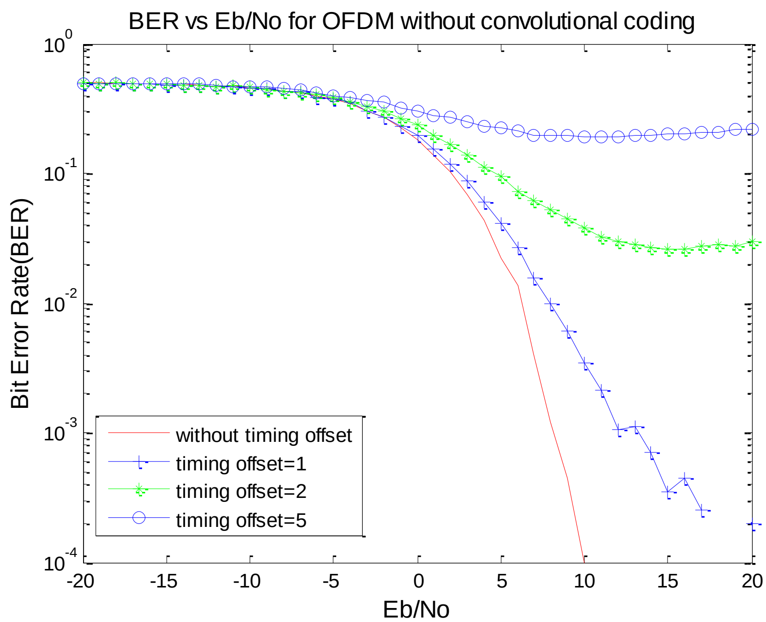

Figure 4. BER vs. Eb/No for OFDM without convolutional coding

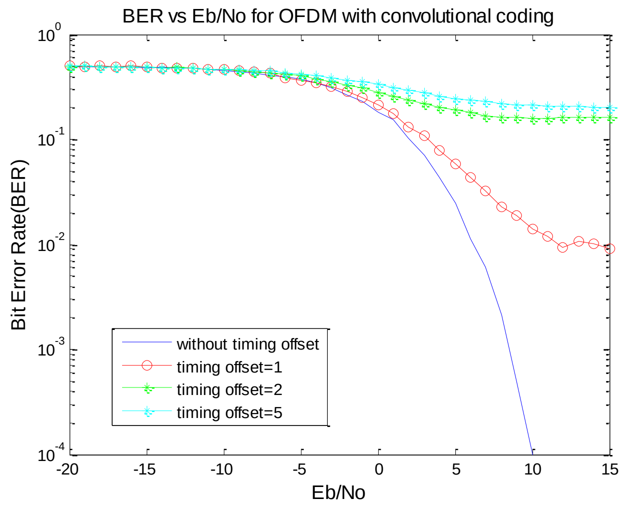

Figure 5. BER vs. E_b/N_o for OFDM with convolutional coding

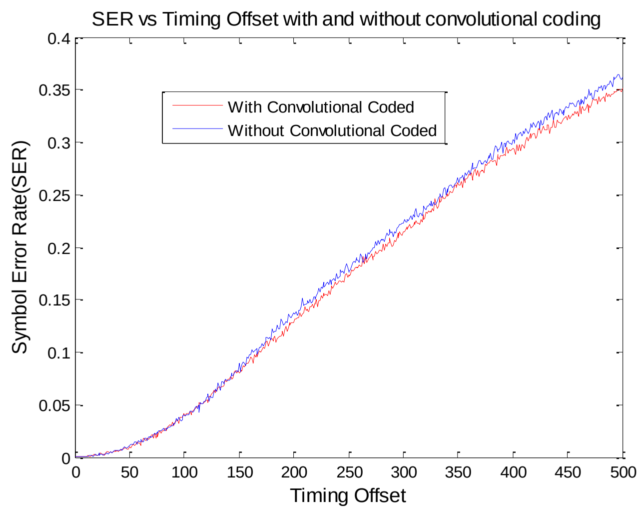

Figure 6. SNR vs. Timing Offset with and without convolutional coding.

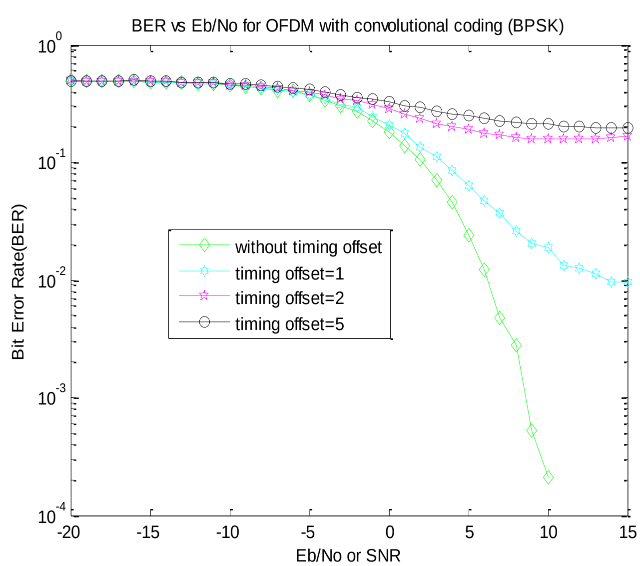

Figure 7. BER vs Eb/No for OFDM without Convolution coding for BPSK

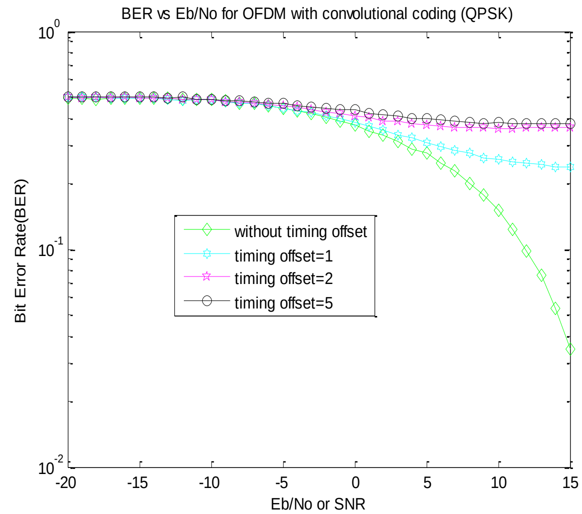

Figure 8. BER vs Eb/No for OFDM without Convolution coding for QPSK

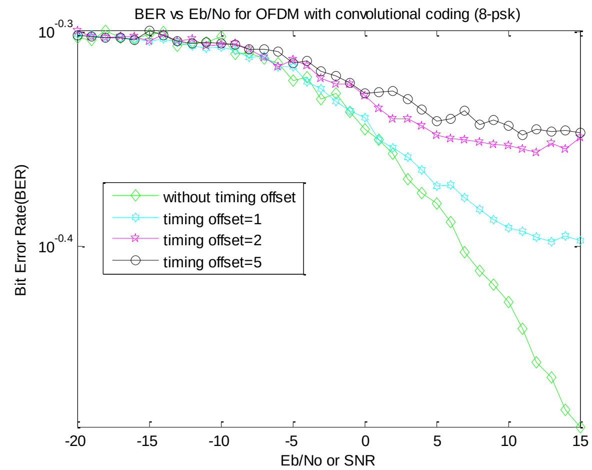

Figure 9. Figure BER vs Eb/No for OFDM without Convolution coding for 8-PSK

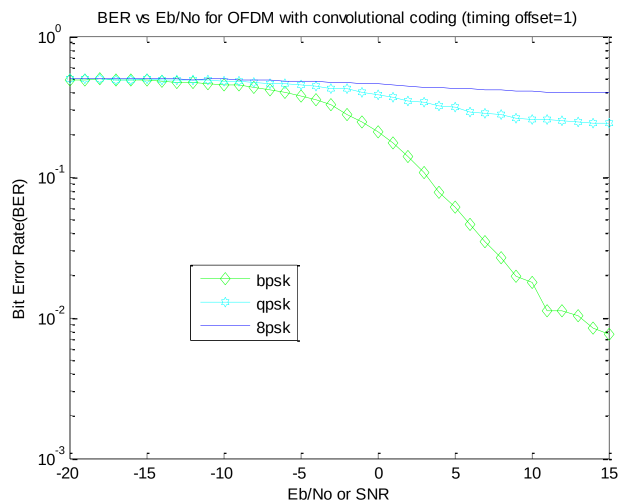

Figure 10. BER vs. E_b/N_o for OFDM with convolutional coding where timing offset=1 for BPSK, QPSK and 8-PSK

Matlab simulation to find the effect on the bit error rate for different modulation schemes where timing offset is present in the system. In this simulation convolution coding is used as channel coding for OFDM system. Fig. 7, 8, and 9 shows the effect on BER due to different timing offset for BPSK, QPSK, and 8-PSK. Fig.10 shows the graph for the system where timing offset=1. The figure exhibits the comparison among the performances of the three popular modulation techniques. Among the three techniques it is seen that BPSK gives the best bit error rate performance for OFDM system with timing offset error.

CONCLUSION

To make a synchronized receiver with the transmitter, there is a need of standard model which will help in removing the effect of timing offset on the symbol stream at the receiver end. The proposed probabilistic method estimates the timing offset error without the use of additional pilots relying on inherent characteristics of the OFDM signal. During the simulation all the natural effects of the frequency channel were carefully maintained. In this paper it is showed using Matlab simulation that BPSK gives the best performance for OFDM system with timing offset error among the three popular modulation techniques- BPSK, QPSK, and 8-PSK. It is evident that particular timing offset equation can be used for particular channels and that can be used in determining the exact length of cyclic prefix to be added to each OFDM symbol to avoid misinterpretation by the receiver. From the analytical curve and Matlab simulated graph it can be seen that the probability of symbol error rate (SER) linearly depends on timing offset. It is expected that further works on the proposed estimated method will lead to a standard model so that the effect of timing offset at the receiving end can be eliminated totally.

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