J. Bangladesh Electron. 11 (1-2); 65-72, 2011

Approximate BER Performance Comparison of Frequency and Time Domain DQPSK/OFDM Systems Over Fading Environment

Mohammed Hossam-E-Haider and Waziha Kabir

Electrical, Electronic and Communication Engineering Department
Military Institute of Science & Technology (MIST), Mirpur, Dhaka

Abstract

The transmission quality in mobile wireless communications is affected not only by additive white Gaussian noise, but also by multipath fading, which drastically changes the amplitudes and phases of wireless signals. This paper proposes the derivation method of the BER in DQPSK/OFDM systems over frequency selective fading channels. In this study the approximate equation of BER with differential coding/differential detection in time domain and frequency domain with delayed waves have been derived. Simulation results confirmed that the proposed approach is applicable to a variety of parameters, such as Doppler frequency shift, delay spread, Rician factor and so on.

Keywords: Bit Error Rate (BER), Time Domain (TD), Frequency Domain (FD), Rician Parameter, Delay Profile, Delay Spread.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is one of the techniques to combat the adverse channel because of its robustness against multipath fading (frequency-selective fading) and high spectrum efficiency. Besides the above appealing properties, however, OFDM systems suffer severe performance degradation from the impacts of Doppler frequency shift and delayed waves caused by multipath transmission because of the co-channel interference, the inter-carrier interference (ICI) and inter-symbol interference in OFDM systems [1]-[3].

This paper proposes the approximate derivation method of the BER in DQPSK/OFDM systems over frequency selective Nakagami-Rice and Rayleigh fading channels (direct wave plus n-ray Rayleigh fading model and only n-ray Rayleigh fading model) in time domain and frequency domain. The equation includes the impacts of Doppler frequency shift and multipath fading by introducing the carrier-to-noise plus interference power ratio (CNIR), which regards these impacts as the Gaussian noise. The proposed approximate equation of BER is confirmed by computer simulation using MATLAB even when the parameters vary.

2. DERIVATION OF BER

The analysis for the impacts of co-channel interference, the inter-carrier interference and the inter-symbol interference (only composite power) on an OFDM transmission has been derived. The approximate equation of the BER in the single-carrier systems of DBPSK is assumed over frequency non-selective Rician fading channels and is given in [5] as

$$P_e = \frac{1}{\dfrac{\{1+J_0(2\pi f_D T_S)\}\dfrac{E_b/N_0}{k_0+1}+1}{\{1-J_0(2\pi f_D T_S)\}\dfrac{E_b/N_0}{k_0+1}+1}+1} \times \exp\left(-\frac{k_0\dfrac{E_b/N_0}{k_0+1}}{\dfrac{E_b/N_0}{k_0+1}+1} \right) \qquad (1)$$

Note that E_b/N_0 is the ratio of energy per bit to the spectral noise density on a Rician (direct plus Rayleigh) wave, k_0 is a Rician factor which is defined as the ratio of the signal power in dominant component over the diffused power, $J_0(x)$ is a Bessel function of the first kind of zeroth order. The Rayleigh wave here means a preceded wave and the Rician factor k_0 is equal to e_{bd}/e_{b_0} in this paper, where e_{bd} is the power of the direct wave and e_{b0} is the power of the preceded wave. The term of $\frac{E_b/N_0}{k_0+1}$ in (1) can be rewritten as follows:

$$\frac{E_b/N_0}{k_0+1} = \frac{E_b/N_0}{e_{b_d}/e_{b_0}+1} = \frac{e_{b_0}}{e_{b_d}+e_{b_0}} E_b/N_0 \qquad (2)$$

which implies the ratio of energy per bit to the spectral noise density on the preceded wave.

The (CNIR) Γ_{NI} involves the carrier-to-noise power ratio (CNR) Γ_N and the carrier-to-interference power ratio (CIR) Γ_I on the preceded wave. In the case of QPSK modulation, the relation between

$\frac{E_b/N_0}{k_0+1}$ and Γ_{NI} is given by

$$\frac{E_b/N_0}{k_0+1} = \frac{\Gamma_{NI}}{2} \qquad (3)$$

A. Impact of Delayed Waves

The transmission channels in this study are assumed to be frequency selective Nakagami-Rice and Rayleigh fading channels as shown in Fig. 1. The total power of the received signal e_b is given in [6] as

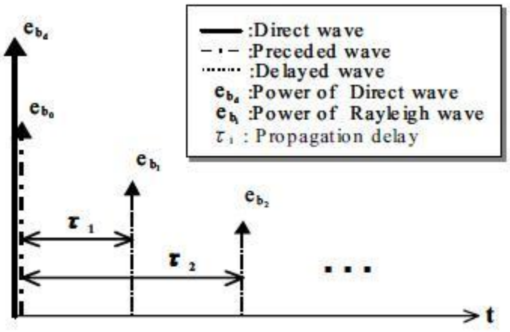

Fig. 1 Model of frequency-selective Nakagami-Rice and Rayleigh fading channels

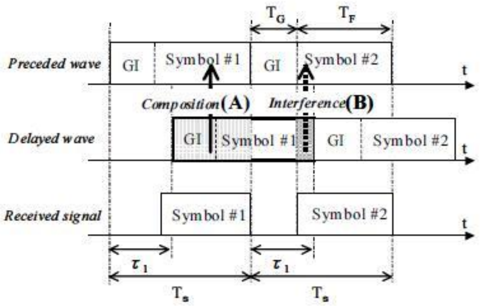

Fig. 2 Composed power and interfered power of one delayed wave

$$e_b = e_{bd} + e_{br} = e_{bd} + \sum_{i=0}^{n-1} e_{bi} \qquad (4)$$

where e_{br} is the total power of all Rayleigh waves. Fig. 2 shows the power e_{b0} and e_{bi} of the preceded and one delayed waves before demodulation. Since a received OFDM symbol is actually a rectangular wave rather than an impulse, the component (A) can be regarded as amplifying the power of the preceded wave if $\tau_i \langle T_S$. The power e_{b0} is composed of the power e_{bi} corresponding to the same symbol #1. Now the composite power is defined as e'_{b_0} and is given in [6] as

$$e'_{b_0} = e_{b_0} + \sum_{for \quad i \ (\tau_i < T_S)} \left(1 - \frac{\tau_i}{T_S}\right) e_{b_i} \qquad (5)$$

In this paper we have neglected the interference component (B) to the preceded wave that comes to be the interference to the symbol #2 as shown in Fig. 2.

B. *Impact of Doppler Frequency Shift*

The co-channel interference power $\bar{I}_{DC}$ and the inter-carrier interference power $\bar{I}_{AC}$ are caused by the impact of Doppler frequency shift that are given in [4] as

$$\bar{I}_{DC} = \frac{(\pi f_D T_S)^2}{2} \qquad \text{(TD)} \qquad (6)$$

$$\bar{I}_{DC} = 0 \qquad \text{(FD)} \qquad (7)$$

and

$$\bar{I}_{AC} = \frac{(\pi f_D T_F)^2}{6} \qquad \text{(TD \& FD)} \qquad (8)$$

Accordingly, (CIR) Γ_{I1} and (CIR) Γ_{I2} on the preceded wave caused by Doppler frequency can be expressed as

$$\Gamma_{I1} = \frac{1}{\bar{I}_{DC}} = \frac{2}{(\pi f_D T_S)^2} \qquad (9)$$

and

$$\Gamma_{I2} = \frac{1}{\bar{I}_{AC}} = \frac{6}{(\pi f_D T_F)^2} \qquad (10)$$

C. *Carrier-to-Noise Power Ratio*

The ratio of energy per bit to the spectral noise density e'_{b_0}/N_0 and CNR Γ_N on the preceded wave is expressed in [6] as

$$\Gamma_N = 2\left(e'_{b_0}/N_0\right) = 2\left(\alpha \frac{e'_{b_0}/e_{b_r}}{k+1} E_b/N_0\right) \qquad (11)$$

$$where \quad \alpha = \frac{T_F}{T_S} = \frac{T_F}{T_F + T_G} \qquad (12)$$

Now, (CNIR) Γ_{NI} on the preceded wave can be expressed as

$$\Gamma_{NI} = \begin{cases} \dfrac{1}{\dfrac{1}{\Gamma_N}+\dfrac{1}{\Gamma_{I1}}+\dfrac{1}{\Gamma_{I2}}} & (TD) \\ \\ \dfrac{1}{\dfrac{1}{\Gamma_N}+\dfrac{1}{\Gamma_{I2}}} & (FD) \end{cases} \tag{13}$$

D. *Approximate Equation of BER*

Accordingly, substituting (3) and (13) for (1) leads the following desired approximate equation:

$$P_e = \frac{1}{\dfrac{\{1+J_0(2\pi f_D T_S)\}\Gamma_{NI}+2}{\{1-J_0(2\pi f_D T_S)\}\Gamma_{NI}+2}+1} \times \exp\left(-\frac{e_{b_r}}{e'_{b_0}} k \frac{\Gamma_{NI}}{\Gamma_{NI}+2}\right) \tag{14}$$

where k_0 in (1) is rewritten as follows:

$$k_0 = \frac{e_{b_d}}{e'_{b_0}} = \frac{e_{b_r}}{e'_{b_0}} \frac{e_{b_d}}{e_{b_r}} = \frac{e_{b_r}}{e'_{b_0}} k \tag{15}$$

3. NUMERICAL EVALUATION AND SIMULATION

The approximate equation is confirmed by comparing the results obtained by the equation with those by computer simulation. Table I shows the system parameters of the OFDM systems. Here, the delay profiling model is based on the exponential attenuation, then the power of each Rayleigh wave e_{b_i} in Fig. 2 is calculated in [6] by

$$e_{b_i} = \exp\left(-\frac{\tau_i / T_S}{S}\right) \tag{16}$$

where S is a delay spread normalized by symbol duration. The delay profiling model is shown in Table II.

Table I: System Parameters

Modulation of subcarrier	DQPSK
Number of FFT point	64
Number of sample point in one symbol duration	64 (GI = 0) 80 (GI = T_S/4)
Number of subcarrier	48
Guard interval (GI)	0, T_S/4
Rician factor, k	3 [dB]
Maximum Doppler frequency normalized by symbol duration $f_D T_S$	0.00, 0.001 0.01, 0.05
Symbol timing recovery	ideal

Table II : Propagation Delays at Each Delay Spread

Delay Spread **[S]**	**Propagation Delays** **[τ_i/T_S]**	**Delay Spread** **[S]**	**Propagation Delays** **[τ_i/T_S]**
$0.01T_S$	0.0125 (=1/80)	$0.04T_S$	0.05 (=4/80)
	0.025 (=2/80)		0.1 (=8/80)
$0.02T_S$	0.0375 (=3/80)	$0.05\ T_S$	0.15 (=12/80)
	0.05 (=4/80)		0.2 (=16/80)
	0.0625 (=5/80)	$0.06T_S$	0.25 (=20/80)
	0.075 (=6/80)		0.3 (=24/80)
	0.0875 (=7/80)	$0.07T_S$	0.35 (=28/80)

Fig. 3 and Fig. 4 show the performance of E_b/N_0 versus the BER obtained from the computer simulation using the derived approximate equations considering parameter of f_DT_S over Rayleigh fading channels in time domain and frequency domain, respectively. Fig. 5 shows E_b/N_0 versus the BER as parameter of f_DT_S over Rayleigh fading channels in time domain and frequency domain on same axis for showing the comparison. In the same way, Fig. 6 and Fig. 7 separately depict the BER performances adding on Rician parameter (k) over Rician fading channels in time domain and frequency domain, respectively and Fig. 8 shows the comparison of the BER performance over Rician fading channels in time domain and frequency domain on the same axis as parameter of f_DT_S. Generally, the faster the f_DT_S is, the more the BER performance degrades. Over Rayleigh fading channels, from Fig. 5 it is found that the derived approximate equations precisely express the characteristics of the above two considered domains and it is found that the BER performance in frequency domain is better than that of in time domain in presence of delay spread, $S = 0.04T_S$. The same is observed for the Rician fading channels shown in Fig. 8. This implies that the influence of Doppler frequency shift during the symbol duration T_S is a critical factor which could severely degrade the BER performance over the frequency selective fading channels in presence of delayed waves. In addition, Fig. 9 shows, as Rician parameter k increases over Rician fading channels, the BER performance will become better. The derived approximate equations can precisely express above mentioned characteristics. The BER derived from the proposed approximate equation coincides with that by computer simulation even when the parameters are varied. Hence, we conclude that the derived approximate equation can precisely express the characteristics of the BER in DQPSK/OFDM systems over frequency selective Nakagami-Rice and Rayleigh fading channels both in time domain and frequency domain.

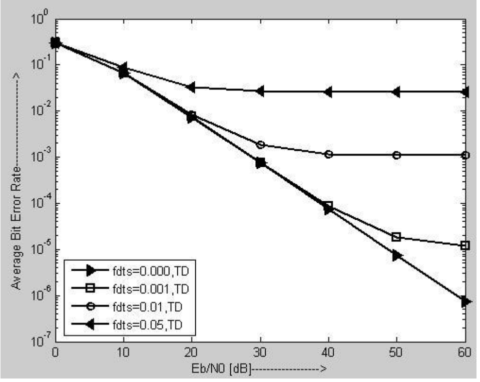

Fig. 3 BER performance over frequency selective Rayleigh fading channels in Time Domain (GI = $T_S/4$, n = 8, S = 0.04 T_S)

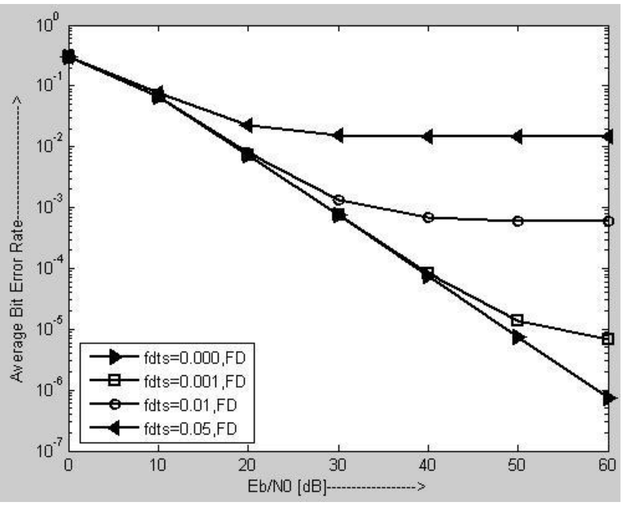

Fig. 4 BER performance over frequency selective Rayleigh fading channels in Frequency Domain (GI = $T_S/4$, n = 8, S = 0.04 T_S)

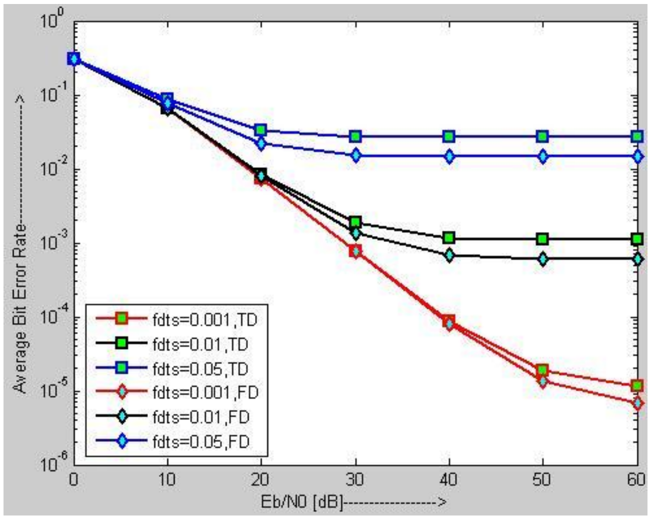

Fig. 5 BER performance over frequency selective Rayleigh fading channels in Time and Frequency Domain (GI = $T_S/4$, n = 8, S = 0.04 T_S)

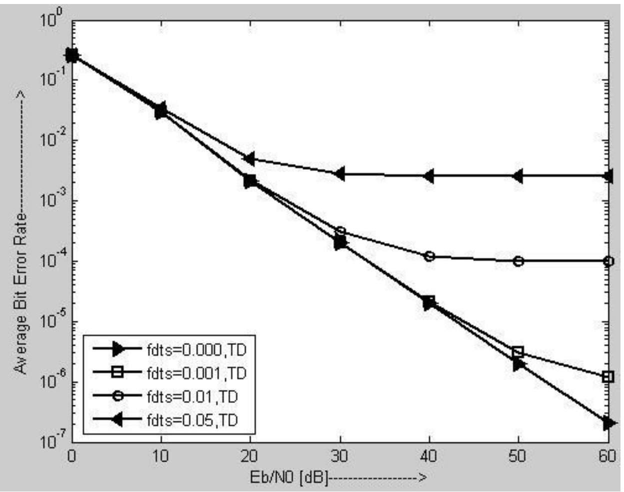

Fig. 6 BER performance over frequency selective Rician fading channels in Time Domain (GI = $T_S/4$, n = 8, S = 0.04 T_S, k = 3 [dB])

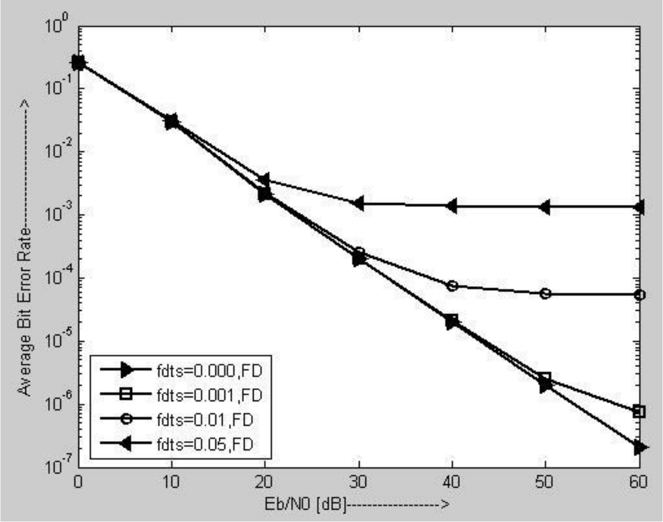

Fig. 7 BER performance over frequency selective Rician fading channels in Frequency Domain (GI = $T_S/4$, n = 8, S = 0.04 T_S, k = 3 [dB])

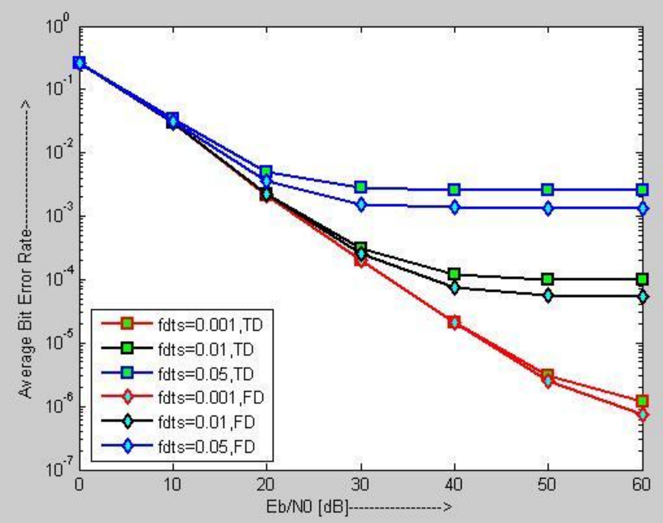

Fig. 8 BER performance over frequency selective Rician fading channels in Time and Frequency Domain (GI = $T_S/4$, n = 8, S = 0.04 T_S, k = 3 [dB])

Fig. 9 BER performance over frequency selective Rician fading channels in Time and Frequency Domain for different Rician parameters (GI = $T_S/4$, n = 8, S = 0.04 T_S, k = 0, 3, 6 [dB], f_DT_S = 0.01)

CONCLUSIONS

An approximate equation of the BER with differential coding/differential detection in the time domain and frequency domain in DQPSK/OFDM systems over frequency selective Nakagami-Rice and Rayleigh fading channels (direct wave plus n-ray Rayleigh fading model and only n-ray Rayleigh fading model) has been proposed. The simulation result revealed that the approximate equation is applicable to a variety of system parameters.

The further direction of this study will be to derive the approximate equation of the BER that includes the impact of the inter-symbol interference power caused due to delayed waves and carrier frequency offset with differential coding/differential detection over frequency-selective fading channels.

REFERENCES

[1] D. Andrea, V. Lottici and R. Reggiannini, “Nonlinear Predistortion of OFDM Signals over Frequency-Selective Fading Channels,” *IEEE Trans. Commun.*, vol.49, no.5, pp.837, May 2001.

[2] B. Yang, K. Letaief, R. Cheng and Z. Cao, “Channel Estimation for OFDM Transmission in Multipath Fading Channels Based on parametric Channel Modeling,” *IEEE Trans. Commun.*, vol.49, no.3, pp.467, March 2001.

[3] U. Tureli, H. Liu, and M. Zoltowski, “OFDM Blind Carrier Offset Estimation: ESPRIT,” *IEEE Trans. Commun.*, vol.48, no.9, pp.1459, Sept. 2000.

[4] F.Sasamori, H. Umeda, S.Handa, F.Maehara, F.Takahata and S.Oshita, “Approximate Equation of Bit Error Rate in DQPSK/OFDM Systems over Fading Channels,” *European Wireless 2002*, vol.1, pp.602-607, Feb. 2002.

[5] F. Sasamori and F.Takahata, “Theoretical and Approximate Derivation of Bit Error Rate in DS-CDMA Systems under Rician Fading Environment,” *IEICE Trans. Fundamentals*, vol.E82-A, no.12, pp.2660-2668, Dec. 1999.

[6] Fumihito Sasamori, Atsuhiro Yamano, Shiro Handa, Fumiaki Maehara, Fumio Takahata, and Shinjiro Oshita, “Approximate Equation of Bit Error Rate in OFDM Systems over Specular Multipath Fading Channels”, *WCNC 2003 - IEEE Wireless Communications and Networking Conference*, vol.3, no. 1, pp. 2096-2101, Mar 2003.