

The Effect of Smoothing Coefficient (ξ) on the Performance Analysis of Alpha-beta-gamma ($\alpha\beta\gamma$) Filter in Radar Tracking

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Abstract

The details of $\alpha\beta\gamma$ filter, a constant coefficient sophisticated smoothing and prediction tracking filter has been demonstrated. The filter is used in sampled data target trackers which can asymptotically track a constant acceleration target. As a polynomial predictor linear recursive filter it can reconstruct position, velocity and constant acceleration based on position measurements. An important sub-class of $\alpha\beta\gamma$ filter is critically damped filter. In this case, the gain coefficients of this filter are chosen on the basis of a smoothing coefficient (ξ). The effect of changing smoothing coefficient for the cases of fast and slow maneuvering targets with and without noise variation factor is observed. Tracker performance is assessed for aforementioned cases on the basis of minimizing the residual error. The optimum range of smoothing coefficient is proposed which results in higher accuracy in tracking.

Keywords: Radar tracker, Alpha-beta-gamma ($\alpha\beta\gamma$) filter, Smoothing coefficient (ξ), Transition matrix (Φ).

1. INTRODUCTION

For nearly three decades the target tracking-trajectory estimation problem has been a fruitful applications area for state estimation. It has found wide applications in both military and commercial areas such as inertial navigation, guidance & control; global positioning system (GPS); differential global positioning system (DGPS); wide area augmentation system (WAAS); inertial navigation system (INS); missiles guidance system; satellite orbit determination; maritime surveillance; air traffic control; freeway traffic system; fire control system; automobile navigation system; fleet management; underwater target tracking system.

Radar has been the device of choice for tracking fast moving targets. Radar is an object-detection system which uses radio waves to determine the range, altitude, direction, or speed of objects. It can be used to detect aircraft, ships, spacecraft, guided missiles, motor vehicles, weather information and terrain. The radar antenna transmits pulses of radio waves which bounce off any object in their path. The object returns a tiny part of the wave's energy to the receiver. By observing the time for a round trip of the pulse, the position (or distance) of the target is calculated. Position of the target is observed for consecutive sample times to predict the next position of the target, i.e., target tracking. Tracking filter is used for this track estimation.

This paper addresses the problem of target tracking in polar measurements. Alpha-beta-gamma ($\alpha\beta\gamma$) filter has been used as tracking filter in our observation. Critically damped filter is used in all of the simulations. Filter gain coefficients are chosen on the basis of smoothing coefficient (ξ). Smoothing coefficient (ξ) is changed to see its effect on radar performance with a view to finding an optimal value of ξ .

2. RADAR TRACKER

A radar tracker is a component of a radar system, or an associated command and control system, that associates consecutive radar observations of the same target into tracks. It is

particularly useful when the radar system is reporting data from several different targets or when it is necessary to combine the data from several different radars or other sensors. With the aid of sophisticated computer systems, multi-function (detection, tracking and discrimination) radars are capable of simultaneously tracking many targets. In this paper, each target is sampled once (mainly range and angular position) during a dwell interval (scan). Then, by using smoothing and prediction techniques future samples can be estimated. Radar systems that can perform multi-tasking and multi-target tracking are known as Track-While-Scan (TWS) radars.

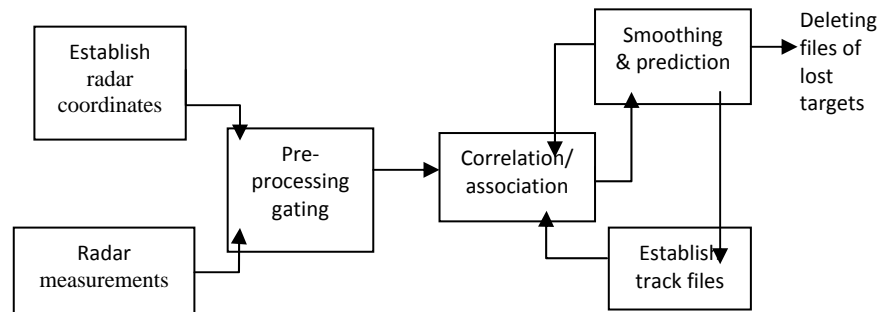


Fig. 1 Simplified block diagram of TWS data processing.

Once a particular target is detected, the radar may transmit up to a few pulses to verify the target parameters, before it establishes a track file for that target. Target position, velocity, and acceleration comprise the major components of the data maintained by a track file. Once a Track-While-Scan (TWS) radar detects a new target it initiates a separate track file for that detection; this ensures that sequential detections from that target are processed together to estimate the target's future parameters. Position, velocity and acceleration comprise the main components of the track file. Typically, at least one other confirmation detection (verify detection) is required before the track file is established. The TWS system places a gate around the target position and attempts to track the signal within this gate. The gate dimensions are normally azimuth, elevation, and range. More precisely, targets must stay within the gate boundary during successive scans.

There are many different mathematical algorithms used for implementing a radar tracker of varying levels of sophistication. However, they all perform steps similar to the following every time the radar updates:

- Associate a radar plot with an existing track (plot to track association).
- Update the track with this latest plot (track smoothing).
- Spawn new tracks with any plots that are not associated with existing tracks (track initiation).
- Delete any tracks that have not been updated, or predict their new location based on the previous heading and speed (track maintenance).

3. TRACKING FILTER

Tracking radar systems are used to measure the target's relative position in range, azimuth angle, elevation angle and velocity. Then by using and keeping track of these measured parameters the radar can predict their future values. Target tracking is important to military radars as well as to most civilian radars. Multi target Track-while-scan radar systems sample each target once per scan interval and filter is used to estimate the target parameters between scans. For this purpose the alpha-beta-gamma ($\alpha\beta\gamma$) filter is commonly used.

3.1. $\alpha\beta\gamma$ Filter

$\alpha\beta\gamma$ filter is a one-dimensional third order, fixed gain, polynomial predictor/corrector linear recursive filter. In simultaneously tracking many target each target is sampled once (mainly range and angular position) during a dwell interval (scan). Then by using $\alpha\beta\gamma$ filter future

samples can be estimated. For radar system range may be an object measured or observed by the $\alpha\beta\gamma$ filter. States of the range are the components of the vector that contains the object and its time derivatives.

For $\alpha\beta\gamma$ filter state vector representing range is given by

$$\underline{x} = \begin{bmatrix} R \\ \dot{R} \\ \ddot{R} \end{bmatrix} \quad (1)$$

Where R, \dot{R} and \ddot{R} are respectively the range measurement, range rate (velocity) and acceleration. In $\alpha\beta\gamma$ filter the state transition matrix assists predicting the next state. The ideal transition matrix elements can be formulated as

$$\phi_{[ij]} = \begin{cases} T^{j-i} \div (j-i)! & 1 \leq i, j \leq n \\ 0 & j < i \end{cases} \quad (2)$$

The state transition matrix for the $\alpha\beta\gamma$ filter is [1]

$$\underline{\phi} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Using the transition matrix the next state is predicted as

$$\underline{x}(n+1|n) = \underline{\phi}\underline{x}(n|n) \quad (4)$$

By inspection the $\alpha\beta\gamma$ filter has Gain matrix

$$\underline{K} = \begin{bmatrix} \alpha \\ \beta/T \\ \gamma/T^2 \end{bmatrix} \quad (5)$$

Zero mean Gaussian noise matrix [2]

$$\underline{G} = [1 \ 0 \ 0] \quad (6)$$

$\alpha\beta\gamma$ filter produces the predicted position and velocity for the (n+1)th observation. It follows an input whose acceleration is constant with zero steady state error. In order to reduce the error at the output of the tracker, a weighted difference between the measured and predicted values is used in estimating the smoothed position, velocity and acceleration.

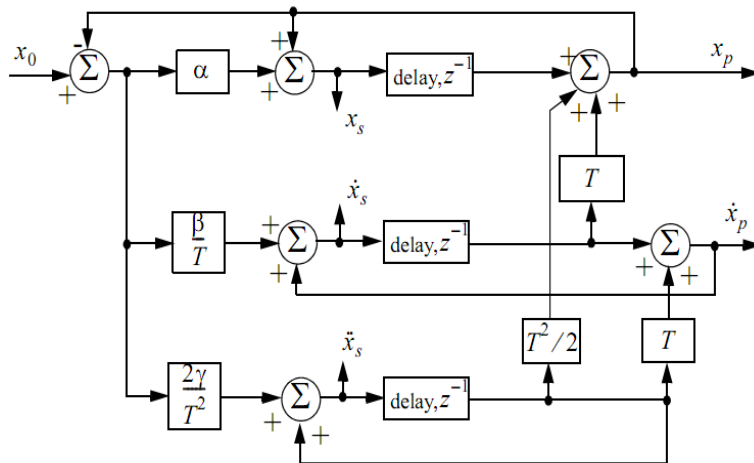


Fig. 2 An implementation for an $\alpha\beta\gamma$ tracker.

The equations for estimating the smoothed position, velocity and acceleration is [2]

$$x_s(n) = x_p(n) + \alpha(x_0(n) - x_p(n)) \quad (7a)$$

$$\dot{x}_s(n) = \dot{x}_s(n-1) + T\ddot{x}_s(n-1) + \frac{\beta}{T}(x_0(n) - x_p(n)) \quad (7b)$$

$$\ddot{x}_s(n) = \ddot{x}_s(n-1) + \frac{2\gamma}{T^2}(x_0(n) - x_p(n)) \quad (7c)$$

$$x_p(n+1) = x_s(n) + T\dot{x}_s(n) + \frac{T^2}{2}\ddot{x}_s(n) \quad (7d)$$

3.2. Smoothing Coefficient (ξ)

An important sub-class of the $\alpha\beta\gamma$ tracker is the critically damped filter, often called the fading memory filter coefficients are chosen on the basis of a smoothing factor (ξ), where $0 \leq \xi \leq 1$. The gain coefficients are given by [2]

$$\alpha = 1 - \xi^3 \quad (8a)$$

$$\beta = 1.5(1 - \xi^2)(1 - \xi) = 1.5(1 - \xi)^2(1 + \xi) \quad (8b)$$

$$\gamma = (1 - \xi)^3 \quad (8c)$$

Where heavy smoothing takes place when $\xi \rightarrow 1$ and no smoothing means $\xi \rightarrow 0$.

4. EFFECT OF ξ ON RESIDUAL ERROR

Gain coefficients α , β and γ are depended on the smoothing coefficient as given in Eq. 8. As the tracker performance is dependent on gain coefficients it is important to maintain such value of ξ for which we can get maximum accuracy in estimating truth position and predicted position.

Simulations have been run to observe tracker performance. Residual error (the difference between truth position and predicted position) has been investigated to evaluate the tracker performance for different ξ . Smoothing coefficient (ξ) has been varied for the four cases mentioned below in two ranges (0.1 ~ 0.9) and (0.9 ~ 0.999) to get a clear view.

4.1. Slow Target without Noise

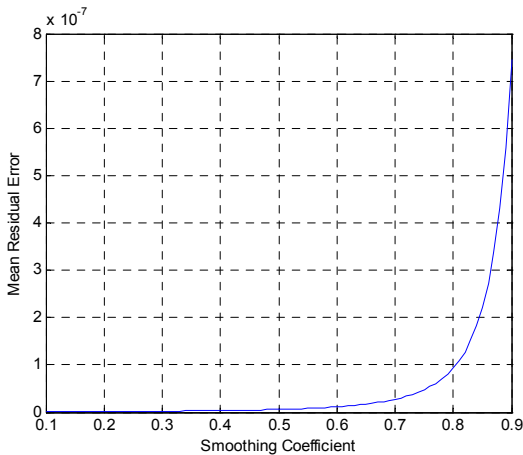


Fig. 3 Residual error vs. ξ (0.1~0.9) for slow target low noise.

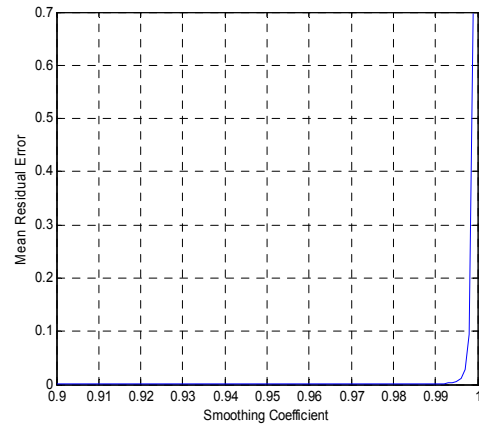


Fig. 4 Residual error vs. ξ (0.9~0.999) for slow target low noise.

4.2. Slow Target with Noise

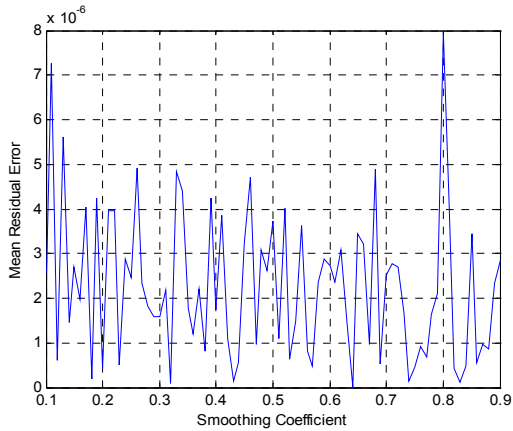


Fig. 6 Residual error vs. ξ (0.9~0.999) for slow target high noise.

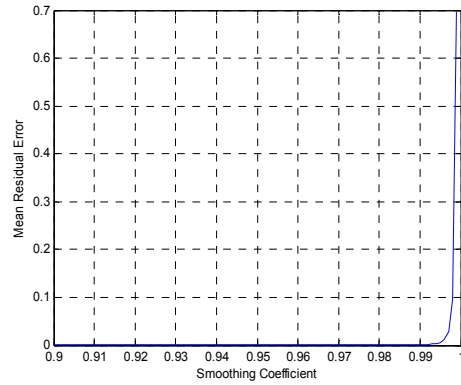


Fig. 5 Residual error vs. ξ (0.1~0.9) for slow target high noise.

4.3. Fast Target without Noise

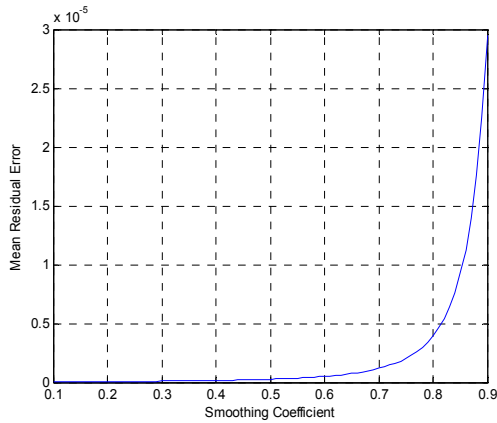


Fig. 7 Residual error vs. ξ (0.1~0.9) for fast target low noise.

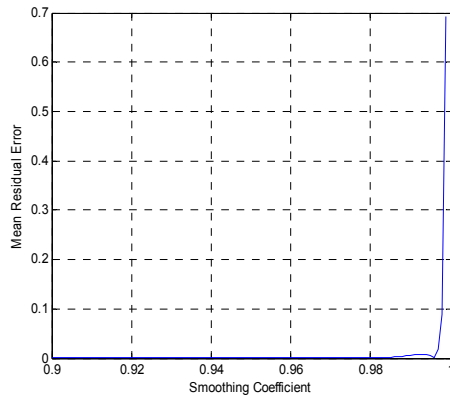


Fig. 8 Residual error vs. ξ (0.9~0.999) for fast target low noise.

4.4. Fast Target with Noise

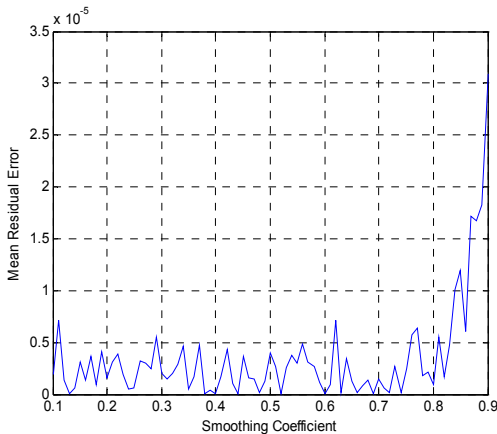


Fig. 9 Residual error vs. ξ (0.1~0.9) for fast target high noise.

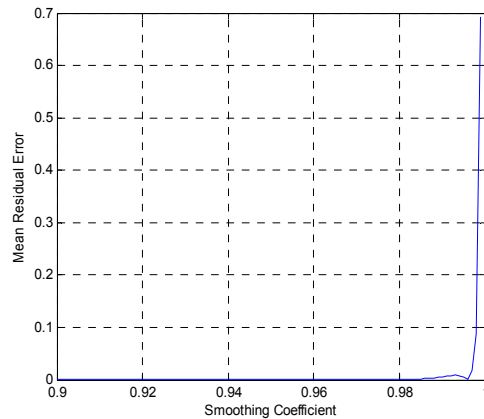


Fig. 10 Residual error vs. ξ (0.9~0.999) for fast target high noise.

4.5. ANALYSIS OF THE RESULTS AND DISCUSSIONS

From Fig. 3 ~ Fig. 6 it is depicted that mean residual error will be minimum for the value of smoothing coefficient upto 0.99 for both with noise and without noise condition for slow moving targets.

From Fig. 7 ~ Fig. 10 it is observed that mean residual error will be minimum for the value of smoothing coefficient upto 0.98 for both with noise and without noise condition in case of fast moving targets.

5. CONCLUSIONS

Smoothing coefficient determines the filter gain coefficients. As the value of smoothing coefficient increases, it causes decrement in the value of gain coefficient which leads to a higher error (residual error) in filter detection. So, the value of smoothing coefficient should follow such an optimum value for which the filter can detect an object of both fast moving and slow moving with higher accuracy. It is proposed that the value of smoothing coefficient should be less than 0.99 for slow moving targets and less than 0.98 for fast moving targets.

As the maneuvering direction, velocity and acceleration of the target is unpredictable it is needed to vary smoothing coefficient as well as gain coefficients to track the object with higher accuracy. Moreover alpha-beta-gamma ($\alpha\beta\gamma$) filter can be used only in tracking targets having constant acceleration. We need a tracking filter which can vary its gain coefficients dynamically to minimize residual error. Kalman filter can serve this purpose.

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